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Scientific report on the research project, stay at Brown University

(April 29th—May 29th 2019)

1. Context

I've stayed at Brown University within the economics department during one month, endowed with a scholarship from LGI. I am grateful to both institutions for having made this collaboration a reality. This report is concerned with the involved scientific project, from preflight design to current state. I will not describe the rest of my life at Brown, which has been full of presentations and talks with other people.

2. The research project – Before the trip

The research project that was designed aimed at building a long-run growth model based on biased technical change – and more precisely on factor-saving technical change, as developed in Senouci (2014) and in Senouci and Barral (2018, extended version forthcoming). The great challenge in long-run growth modeling is to account for structural breaks by using the smoothest ingredients possible. This is not a small challenge. For instance, the Unified Growth Theory ('UGT', Galor & Weil (2000)) needs very specific assumptions to yield a demographic transition.[1] The prime idea is that a change in the direction of technical change is likely to yield a qualitative change in the growth path, i.e. a change of growth regime. The assumption on factor-saving technical change might also bring an explanation to the evolution of factor shares – namely, the secular decrease in the land and unskilled labor shares of income, and the secular increase in the physical and human capital shares.[2]

The research objective is thus to build a model simpler than UGT and that rests on the least ad hoc assumptions possible. For this latter reason, I've decided to work in an evolutionary framework rather than in a neoclassical framework, which is the second building block of this project. Indeed, evolution is a robust paradigm where technical change and preferences are the outcome of a selection process. This class of models has the virtue of minimality and also speaks to the broadest literature on animal and human evolution.

The two ingredients of my approach are biased technical change (to induce regime-switching) and natural selection (to need only minimal assumptions, and for the sake of theoretical robustness).

3. What has been done

I have spent my research time at Brown doing two things.

Firstly, I attempted to build a general framework to represent a selection mechanism on technology and preferences. This model should not be thought as the definitive one, but rather as its backbone. I wanted a closed mathematical structure able to answer the following question *"for a family of production functions $(F_i)_{i \in I}$, which one survive at equilibrium? What's the associated strategy from the part of the surviving agents?"*

On the technological side, it is clear that if $\exists i, j \mid F_i > F_j$, i.e. F_i dominates F_j, then F_j cannot survive. However, the problem is richer when the production functions do not dominate each other, which is exactly what happens

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[1] Namely: human capital h enters the production function as labor-augmenting productivity, there also exists one index of land-augmenting productivity A, with the two terms in interaction (through $h_{t+1} = h(e_{t+1}, g_{t+1})$, with $g_{t+1} = A_{t+1}/A_t - 1$). UGT also involves a subsistence consumption constraint, and a 2-period log utility function. g_{t+1} increases as population increases like in endogenous growth theory *à la* Romer.

[2] Of course, the statement according to which modern economic growth comes with an increase in the share of accumulable factors (physical & human capital) at the expense of the share of non-accumulable factors (land, unskilled labor) might be wrong for some places and some times, but is indisputable over the very long run.

with factor-saving technical change. Imagine that at date 0, only one production function exists, F and that the economy is at steady state where inputs are at R^*. At date 1, several new production functions F_i emerge. The factor-saving assumptions consist in assuming that $\forall i, F_i(R^*) = F(R^*)$ and that some F_i's might be more efficient than others at different input levels. For instance, in a two-input (capital & labor) framework, labor-saving (capital-using) technical change results in a function F_i that yields more output than F for higher capital-labor ratios, while capital-saving (labor-using) technical change results in a function F_i that yields more output than F for lower capital-labor ratios.

On the preferences side, the selection mechanism translates into the assumption that households and firms make optimal choices at steady state – otherwise, another household or another firm would overcome them and higher fitness means prevalence at equilibrium. Thus, for instance, technology is used efficiently given marginal costs and productivities, and the fertility behavior of the agents should be consistent with maximum steady-state fertility. To the best of my knowledge, such a framework does not exist in the literature.

Secondly, I thought about what facts I should try to explain. As Oded told me, we always want models able to explain a lot of things, but the reality is that it is more fruitful to decompose the theoretical effects that we want, and to try to generate them one by one. So I tried to be clever and think about the broadest features of long-run growth phenomena that I wanted to explain.

When long-run growth economists do that, they most probably land back on the Holy Grail of the profession, the demographic transition (DT). The DT is a phenomenon which is peculiar to the human race and which implications are far-reaching. We can hardly imagine modern growth of real GDP per capita without the DT, and we can hardly deny that the DT probably has some effect on technical change (rate or direction).

This was not very original, and that did not give me a hint on what could provoke the DT. As is well-known, it is easy to write down Malthusian models, and the Barro-Becker framework explains how an increase in income might result in lower fertility; but it is also notorious that to have a model that features a transition between these two regimes is very tricky – probably only UGT does that. So I looked for a cause that might trigger the DT, a slow and long cause that might credibly trigger a DT when some variable passes some threshold.

4. Outcome & next steps

The outcome of the first part was a very simple framework. The model remains in the purest Darwinian spirit, with the replicators being the people themselves. I present it here quickly.

The only ingredients are (a) the family of functions considered, and (b) the cost functions for children. We consider a three-input framework, with labor, capital and land. Land is in fixed quantity X. Type i population is L_i and total population is $L = \int_i L_i$.

People live for two periods. In the first period, they inherit capital from their parent. All people of all types face the same land-population ratio, as land is a common good. Young people work with the capital they inherited. Agent i is able to produce $f_i(k_i, x)$ if she inherited k_i units of capital and if the global land-population ratio is at $x = X/L$. The cost of one child is ρ units of the good. The cost function for capital is $C(nk)$ units of the good, where n denotes the number of children and where k denotes capital per child. C is assumed to be strictly increasing and strictly convex.

Let's take the point of view of agent i, and let's assume that the economy is at steady state with no technical change, at some $L^* = X/x^*$. Then, f_i survives equilibrium if and only if:

$$n_i^*(x^*) \equiv \begin{pmatrix} \max\limits_{(n_i,k_i)} & n_i \\ \text{s.t.:} & f_i(k_i, x^*) \geq \rho n_i + C(n_i k_i) \end{pmatrix} \geq 1$$

If $n_i^*(x^*) < 1$, function f_i does not survive steady state x^*. If $n_i^*(x^*) = 1$, f_i is exactly just compatible with steady state x^*. If $n_i^*(x^*) > 1$, then f_i beats the other production functions, which makes x^* not compatible with a steady state.

All in all, we can characterize the effective steady state as the lowest possible x^* for which there exists some $i \mid n_i(x^*) = 1$ – given that, then, other states $x < x^*$ won't be steady states as $n_i(x)$ will be < 1 for these lower land-population ratios and so population will tend to decrease, and x to increase. Then, surviving production functions are the ones for which $n_i^*(x^*) = 1$ exactly. Within this framework, it is straightforward to simulate steady-state-to-steady-state dynamics. Technical change opportunities can be summarized to the polar cases, which makes the approach indeed both easier and less ad hoc than the induced innovation literature *à la* Kennedy (1964) for example, where the curvature of the innovation possibility frontier is the ultimate driver of the direction of technical change. Here, and like in Senouci (2014), it is often the case that only polar choices of innovation are admissible, which keeps the analysis tractable. Depending on the choice of function $C(\cdot)$, on the functional form of the production functions and on technical change opportunities, this model is able to deliver original dynamics.

However, this model cannot yield a DT. Since the selected i's are the ones with the higher n, there is no point in restricting fertility at steady state. I believe that to yield a DT in this framework would require some additional ingredient, on which I don't want to speculate right now.[3]

On the second question, I have identified a variable that is a good candidate to be the trigger of the DT. That's the land share. According to Allen (2009), the land share has decreased steadily all over the 1st and 2nd Industrial Revolutions, from more than 20% of GDP in 1770 to less than 5% of GDP in 1890, at the time of the European DT. According to Piketty and Zucman (2014) data, land value was 358% of GDP in 1750, and only 119% of GDP in 1885 and 52% of GDP in 1901. The secular fall of the labor share is a phenomenon that we can easily understand within the model above; since everybody tries to increase its asymptotic fertility, everything else equal technical innovations that are land-saving innovations are selected when competing against land-using or land-neutral innovations, which do a worse job at adapting to a lower land-population ratio.

On the theoretical side, there's also a reason to expect the DT to be more likely when the land share is at a low level. In theory, a lower land share reflects a lower elasticity of output with respect to land, which we can think of as the consequence of a land-saving bias in technology. DT is best thought as the consequence of an increase in the capital share (human, but also physical) and a decrease in the unskilled labor share. Children become less valuable unless they are endowed with much capital, provoking a relative decrease in fertility. To innovate in a capital-using direction requires taking one more roundabout than other innovations: one must innovate in a capital-using direction, then accumulate capital before the effects of the innovation are materialized. In contrast, a capital-saving innovation bears its fruits more quickly as an economy on capital has an immediate positive effect on the budget constraint. The patient strategy might be beaten by the impatient strategy simply because of that kind of effect: impatient strategies 'occupy the field' before the patient strategies, which cannot catch up at the new and lower land-population ratio. When the land share is low, this effect is diminished and the patient strategy's relative score increases.

To summarize the state of the project, the minimal backbone seems solid and flexible enough to be adapted to several assumptions. We can easily understand why the labor share would go down in a Malthusian economy. It is however hard to give rise to a DT within this "$\max n$" framework, and my next step on this project is clearly to find an assumption able to give rise to an evolutionarily optimal decrease in fertility.

A biological anthropologist told me, there seems to exist no evolutionary framework to represent the passage from optimal quantity strategies to optimal quality strategies, featuring a lower net fertility level at the new equilibrium. This work tries to fill this gap.

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[3] To yield a DT in the "$\max n$" framework seems to require to assume that some emerging costs make optimal n lower, and endowment per child higher. An negative externality due to population might do (making the budget constraint $f_i(k_i, x^*) - e/x^* \geq \rho n_i + C(n_i k_i)$, with e being the size of the externality). It is not sure whether introducing globalization and a periphery would work.